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A Bayesian Approach to the Estimation of Metal Flows in Waste Streams

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Abstract

Metal contents in waste are normally presented without any measures of uncertainty, and, if such measures are given, they are normally based on subjective judgements. Moreover, it is difficult to analyse existing data sets by employing traditional statistical procedures. The number of analysed samples can be very small and the sampling strategy is often unknown. Outliers are common and it is not unusual that the standard deviation is larger than the mean.

In this paper, we propose a method that combines subjective judgement of uncertainty with actual observations to establish uncertainty bounds. The Bayesian statistical framework is used to set up a probability model that reflects our scientific problem, and this provides a theoretical basis for the inference problem.

1. Introduction

Substantial efforts are made to reduce the amount of wastes that needs to be dumped in landfills. However, extraction and utilisation of materials and energy from waste streams causes new substance flows that may have undesirable effects. Hence, the presence of metals and other substances in wastes that are recycled or incinerated calls for a thorough analysis. Also, there is a need for a systematic handling of the large uncertainty that is typical for almost all data regarding waste-related substance flows.

Examination of already published reports shows that metal contents in wastes are normally presented without any measures of uncertainty and, if such measures are given, they are normally based on subjective judgements. Moreover, it is difficult to analyse existing data sets by employing traditional statistical procedures. The number of analysed samples can be very small and the sampling strategy is often unknown. Outliers are common and it is not unusual that the standard deviation is larger than the mean.

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In this article, we show how a Bayesian statistical framework (Gelman *et al.*, 1997) can be employed to combine actual observations with prior knowledge or expert judgements of uncertainty bounds. Furthermore, we show how we can handle the fact that wastes are typically very heterogeneous and exhibit a substantial variation between suppliers and deliveries. The general principles proposed are illustrated with data regarding heavy metals in waste wood used for energy production in Sweden.

2. Data

The data on metal concentrations in Swedish waste wood were collected at Igelstaverken in Södertälje, Sweden. The histogram below shows the distribution of the concentrations of arsenic in the sample.

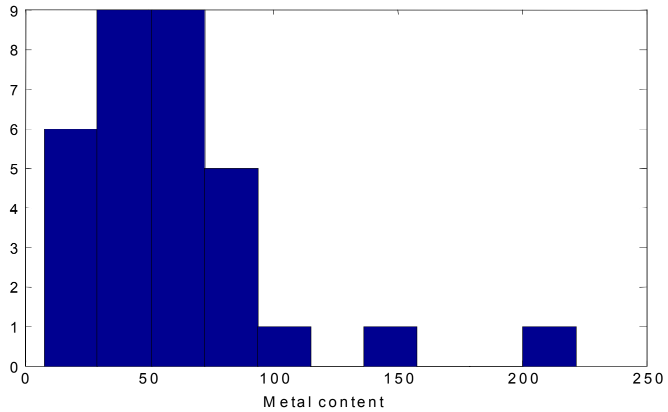

Figure 1. The distribution of the arsenic concentration in the sample.

3. A Bayesian statistical framework

3.1 Single-parameter model

Prior information or subjective judgement of uncertainty can be described mathematically by a prior distribution as is shown in figure 2.

Once the prior distribution has been selected, it must be updated with the information in the data. The updating procedure ultimately ends up in a posterior distribution that reflects the uncertainty in the estimate of the mean metal content in the waste wood.

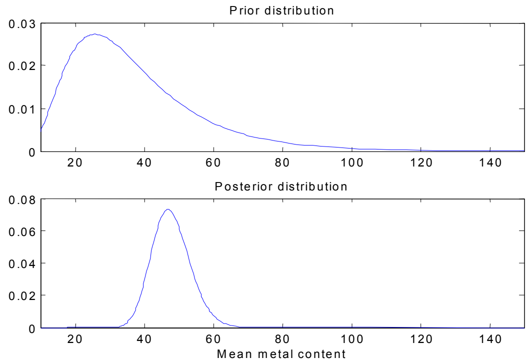

Figure 2. Combining prior information with data to form the posterior distribution.

3.2 Hierarchical model

In the following, we assume that wastes that are extracted for energy production are delivered to a single power plant by different suppliers or from different sources. This implies that the data that can be collected have a hierarchical structure that is illustrated in figure 4. A Bayesian hierarchical model implies the following: (i) we choose a prior distribution of the so-called hyper parameters, μ and σ; (ii) the parameters, μ_i and σ_i ,which characterize the metal content in wastes from different suppliers are viewed as a sample from the super population; and, (iii) given the observed values, we compute the posterior distribution of all the parameters. Computation of the posterior distribution is most conveniently performed by simulation.

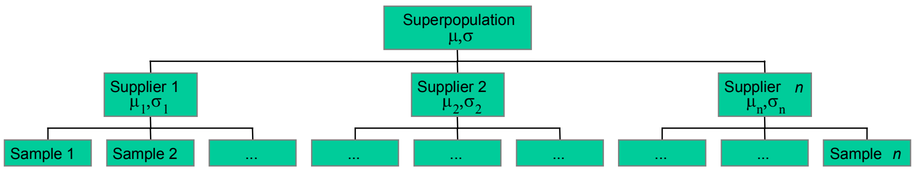

Figure 3. Hierarchical structure of data representing metal contents of wastes delivered by different suppliers to a single power plant.

Considering that the metal content in wastes can vary considerably between suppliers and that outliers are often present in collected data, we propose that the prior distribution is assumed to have a heavy tail. To be more precise, we used an extreme value distribution to describe the prior. Posterior and predictive posterior distributions were then calculated by using MCMC (Markov Chain Monte Carlo) simulation

(Gilks *et al*., 1996). This yields histograms that show the posterior distributions of the mean metal content in the waste wood from the different suppliers. A histogram of the mean arsenic content in wood from supplier 1 is shown below.

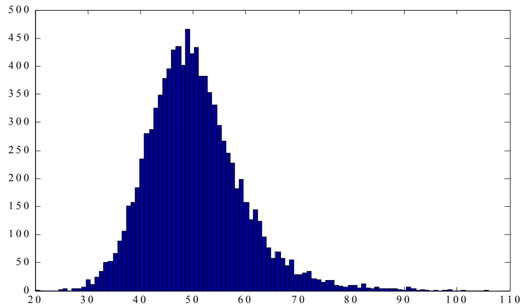

Figure 4. Diagram of the mean arsenic content in wood from supplier 1.

The results obtained for the metal content in waste wood from buildings showed that the hierarchical structure was appropriate, although the large variation between different deliveries from the same waste supplier sometimes overshadowed the variation between suppliers. Furthermore, the case study illustrated how MCMC simulations can be used for three different purposes: (i) retrospective analysis of the metal content of the wastes that have been abstracted, (ii) predictive analysis of the metal content in future deliveries from existing suppliers, and (iii) predictive analysis of the metal content of wastes from a new supplier drawn from the super population.

4. Conclusions

A Bayesian approach is attractive, because it enables uncertainty assessments that are based on a combination of expert judgements and empirical data. Furthermore, it produces uncertainty bounds regardless of the amount of data that is available. The hierarchical model proposed here plays an important role in structuring the collection and analysis of the data. Any substantial difference between suppliers will also produce uncertainty bounds that are more realistic than those obtained by non-hierarchical models.

Bibliography

Gelman, A., Carlin, J.B, Stern, H. S., Rubin, D.B. (1997). *Bayesian Data Analysis.* New York: Chapman & Hall.

Gilks, W.R., Richardson, S., Spiegelhalter, D.J. (1996) *Markov Chain Monte Carlo in practice*. London: Chapman & Hall.