EnviroInfo 2007 (Warschau)
Environmental Informatics and Systems Research

Game Theory Based Water Quality Models for Reservoir Management

Shouke Wei[1], Albrecht Gnauck[1]

Abstract

The management of reservoirs is very complex, and it is usually involved with multi-users. Conflicts usually arise between multi-users due to their competing for water use. Game theoretic models and notions can be applied to model such problems and find solutions. In this paper, water quality management of Danjiangkou Reservoir in Hanjiang River Basin in China is analyzed and modeled as a game. The results show that non-cooperation will result in a prisoner's dilemma and exacerbate the deterioration of water quality. For the case of Danjiangkou Reservoir, compensation is proposed as the best solution to solve the prisoner's dilemma.

1. Introduction

Reservoir is a complex ecosystem, which provides a variety of services (consumptive and productive). However, degradation of water quality has become one of the important elements resulting in water scarcity in many countries (Wang/Fang/Hipel, 2003; Wei/Gnauck, 2007). It is widely recognized that the effect on water quality comes from human activities more than that from the internal ecosystem. Water quality management is usually involved with interactive and interdependent multi-actors with contradictory or conflicting interests (van der Veeren and Tol, 2003), goals and strategies. In the absence of market and exclusive property rights, conflicts between the multi-actors competing for water are unavoidable. Wei and Gnauck (2007) stated that the traditional controlling instruments do not work well in solving such conflicts. In this paper, game theory is applied to model such conflicts, and a case study on Danjiangkou Reservoir in Hanjiang River Basin in China is analyzed to illustrate the game modelling approach.

2. Methodology and data

2.1 Methodology

Water quality management is modeled within the framework of game theory. The game modeling process consists of defining the game, solving the game and interpreting the results. Non-cooperative and cooperative game approaches are used separately to model and simulate conflicts of water quality. In order to formulate the payoff functions of the players, statistical and econometric regression methods are applied.

2.2 Data collection

All the data are collected from monitoring stations, official reports, planning and Chinese yearly books. The main types of data include socio-economic data (added values of industry and agriculture, animal

[1] Department of Ecosystems and Environmental Informatics, Brandenburg University of Technology Cottbus, Knrad-Wachsmann-Allee 1, D-03046 Cottbus, Germany, Phone: 0355/692713, Fax: 0355/682743,
emails: sigmundwei@yahoo.com, Albrecht.Gnauck@tu-Cottbus.de, internet: http://www.tu-cottbus.de/umweltinformatik/

husbandry, waste water discharge), hydrological data (annual inflow of Danjiangkou Reservoir) as well as water quality data on Danjiangkou Reservoir in Hanjiang River Basin.

3. Danjiangkou Reservoir in China

The U-shape Danjiangkou Reservoir is located in the Hanjiang River Basin. It covers an area of 1050 km^2 and has a total storage capacity of 17.45 billion m^3 with a normal pool level of 157m. It services as flood prevention, electric power generation, water supply for production, irrigation, navigation and drinking. In general, the variation of the water level in front of the dam is 18 m; the biggest water depth in front of the dam is about 90 m. Based on the data of 1956 - 1998, the means annual precipitation above Danjiangkou is 890.5 mm and average natural inflow volume into the Reservoir is 38.78 billion m^3. From 2010 Middle Route of South to North Water Transfer (MRSNWT) Projects will transfer water from this reservoir to north of China in order to alleviate the severe water scarce problem there. The water transfer requires that water quality should meet the water class II of Chinese Environmental Quality Standards for Surface Water (GB 3838—2002) in 2010. Thus, the cities in the reservoir catchment are required to treat the waste water and to reduce their waste discharge.

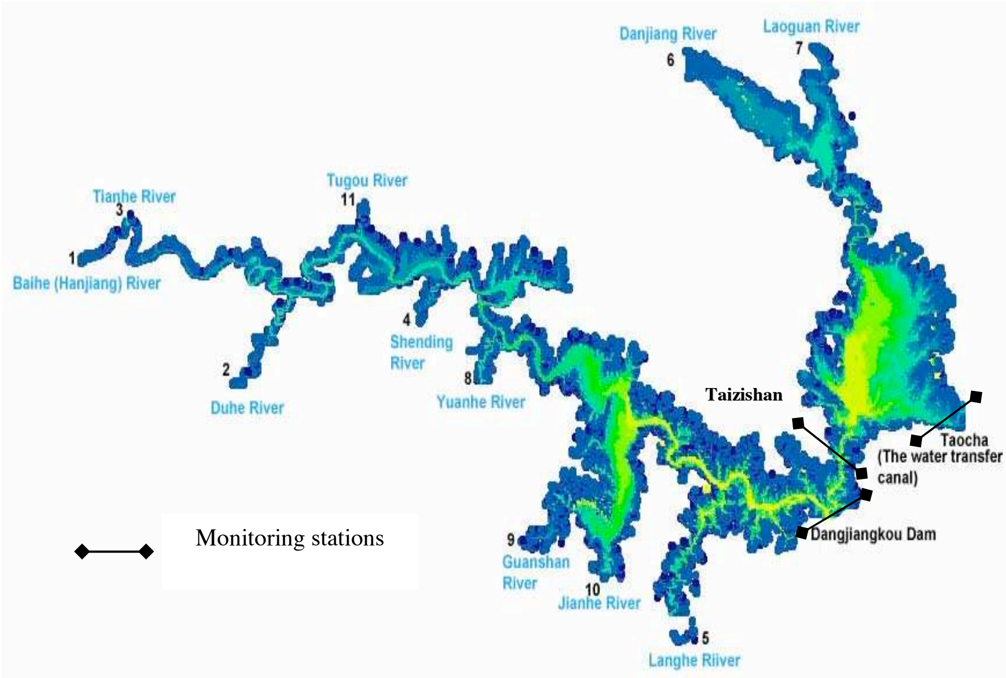

Figure 3.1: Sketch of Danjiangkou Reservoir and three monitoring stations

The monitoring stations - Dam, Taizishan, Taocha– are selected as the data collection pots (Figure 3.1). Concerning the concentrations of total nitrogen and the total phosphorus, the reservoir is in a mesotrophic condition. The monitoring data from 1995 to 2004 show that the water deterioration of the reservoir is characterised by the increase of TN (Figure 3.2). Except the high concentration of TN, the water quality of the reservoir conforms to the water class II of Chinese Standards.

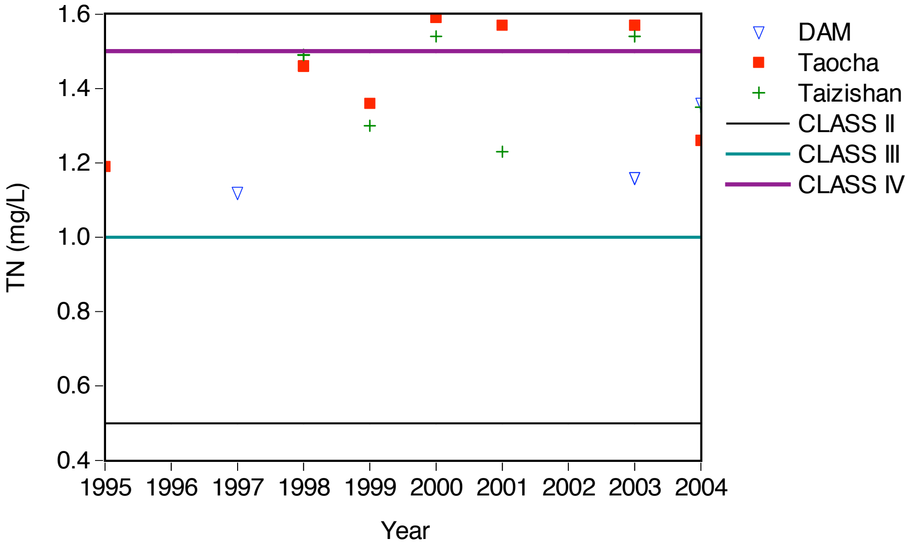

Figure 3.2: TN concentrations of three monitoring station in Danjiangkou Reservoir from 1995 to 2004

4. A game theory approach of water quality

A game is a metaphor of the rational behaviors of multi-actors in an interacting or interdependent situation, such as cooperating or coalition, conflicting, competing, coexisting, etc. Game theory has two broad approaches. Non-cooperative approach deals largely with how individual players interact with each other to achieve their own goals in the situation of lacking binding agreements, whereas a cooperative approach deals with what coalitions form they can use and how the available payoff they can divide or allocate among them. (Fundenberg/Tirole, 1996).

4.1 A game model

4.1.1 Assumptions

The game is finite, static with complete information. All the players are rational, and their aim is to maximize their welfare. There is no intervention of administration during game processing, but the game processing is influenced by the current policies. All data are authentic. Pollutant discharge and economic values growth are correlated following a curve like inverse U.

Figure 4.1.1 illustrate the correlation between pollutant discharge and economic value growth. When the production activities start, the economic value is produced and pollutants are also discharged. In the interval of [0 P_{iq}), V_i is growing with P_i increasing. However, V_i reaches the maximum value of V_{iq} at P_{iq}.

If P_j is still increasing, V_i will decrease. When the amount of P_j overpasses the threshold, say maxP_j, the water body has lost its function and V_i will be zero because of no water for production.

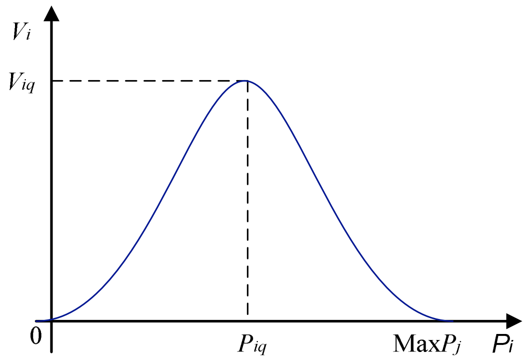

Figure 4.1.1: The correlation between pollutant discharge and economic value production

4.1.2 Formulating the game model

Waste discharge or pollution is a public bad, where it is a free riding problem. Every polluter does not want to reduce its waste discharge or treat the waste water, because it can be better off by free-riding others' treatment. This problem is modelled as a game of Prisoner's Dilemma, in which cooperation brings a better result but each player prefers non-cooperation to free ride whatever the other players do.

Suppose there are n cities which discharge waste water into the reservoir. The process of pollutant j into the reservoir can be classified as pollutant production, discharging it into the river, reaching into the reservoir. Some of the pollutants will be decayed due to biochemical processes. This process is expressed as follows:

$$M_{ij}^{t} = \sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij}^{t} \lambda_{ij} k_{ij} \varphi_{ij} \tag{4.1.1}$$

where M_{ij} – load of pollutant j discharged by player i into the reservoir, t – different periods of time, P_{ij} – amount of pollutant j produced by player i, λ – the rate of pollutant j into the river, k – rate of pollutant j into the reservoir, and φ – loss rate of pollutant j in the reservoir.

The annual average concentration of the pollutant j reached in the reservoir is expressed as follows:

$$C_{ij} = M_{ij} / Q_f \tag{4.1.2}$$

where C – annual average concentration of pollutant j reaching in the reservoir, Q_f – annual natural inflow of the reservoir.

The game is defined as follows:

$$G = \langle N_i, S_i, U_i \rangle \tag{4.1.3}$$

where $N_i = \{N_1, N_2 \ldots N_k\}$ is the set of players, S_i is the strategy of every player i, U_i is the payoff function of every player i.

The three players are $N_1 = \{C_1, C_2, C_3\}$; $N_2 = \{C_4, C_5\}$; $N_3 = \{C_6\}$, and C_1, C_2, C_3, C_4, C_5 and C_6 are cities in the reservoir catchment (Figure 4.1.2).
The strategies for every player N_i is to choose the amount of pollutant j to reduce, i.e.

$$S_i = P_{ij} \in S_i = [0, \infty) \tag{4.1.4}$$

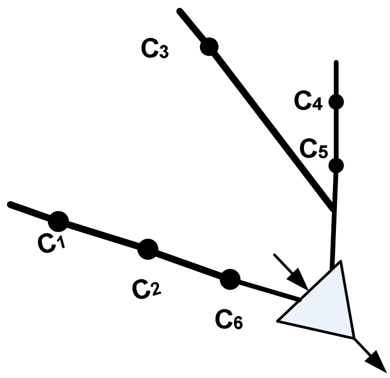

Figure 4.1.2: The sketch of the cities around or above the Danjiangkou Reservoir

The payoffs function of every player is expressed by:

$$U_i(P_j) = V_i P_j - c_i P_j \tag{4.1.5}$$

where $U_i(P_j)$ – payoff function of play i to produce amount of pollutant j, V_i – economic value to produce per unit of pollutant j, c_i – cost to reduce per unit of waste.

The Nash Equilibrium of $(P_1^*, P_2^* \ldots P_n^*)$ is given by the fact that every player Ni will maximize its payoff, given that other players choose $(P_1^* \ldots P_{i-1}^*, P_{i+1}^*, P_n^*)$. Therefore, to solve the equation (4.15), the first order condition (FOC) is valid:

$$V_i P_i^* + P_i V_i' - c = 0 \tag{4.1.6}$$

Therefore, the collective waste discharge (P) should be the sum of n players, i.e.

$$VP^* + \frac{P^* V'}{n} - c = 0$$

$$P^* = \frac{nc}{nV + V'} \tag{4.1.7}$$

In contrast, if all the players cooperate with each other, they will solve the social optimum P^{**},

$$MaxU(P) = VP - cP \tag{4.1.8}$$

Then the first order condition (FOC) is valid:

$$VP^{**} + P^{**}V' - c = 0 \text{ ,}$$

$$P^{**} = \frac{c}{V' + V} \tag{4.1.9}$$

Comparing equations (4.1.7) and (4.1.9), it gets:

$$P^{*} > \mathrm{P}^{**} \tag{4.1.10}$$

The result of equation (4.1.10) proves that the players maximize their payoff while the society gets environmental losses. Therefore, the deterioration of water quality in the Danjiangkou Reservoir is caused by the non-cooperative behaviors of the players themselves. This game model is a very useful tool to analyze the conflicts between individual rationality and collective rationality.

4.2 TN reduction game of Danjiangkou Reservoir

Since the high concentration of TN is the main problem of water quality of Danjiangkou Reservoir, water quality management in this study refers to TN reduction. The model of annual average TN concentrations (Figure 4.2.1) is give by:

$$\mathrm{TN} = 0.160{*}D_t{*}T - 319.318{*}D_t - 0.093C(3){*}T + 187.732 + [\mathrm{MA}(1),\mathrm{INITMA}] \tag{4.2.1}$$

$$D_t = \begin{cases} 1 & (1995-2000) \\ 0 & (2000-2004) \end{cases}$$

where MA(1) = -2.122, INITMA – 1995 with $R^2 = 0.965$ and adjusted $R^2 = 0.938$.

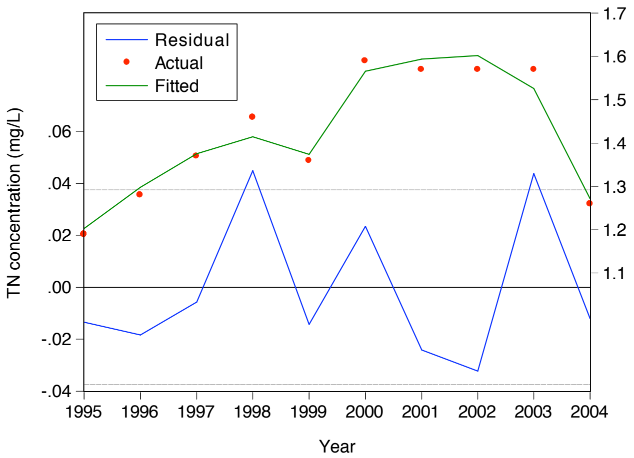

Figure 4.2.1: Model fitting of annual average TN concentrations of Danjiangkou Reservoir

If all the players acted via non-cooperation cooperate with each other, i.e. without water transfer, the TN concentrations would follow the trend of 1995 to 2000. However, all the players have to reduce their

TN pollutants because of governmental policy. In order to compare the results of the game with these two scenarios, the model is used to predict the TN concentrations up to the year of 2010.

5. Results and Discussion

The results show that the players would choose non-cooperation if there was no water quality treatment policy. This non-cooperative game will aggravate the water quality in the reservoir (Figure 5.1a). However, water transfer project requires improving the water quality to conform to class II. The modeling result (Figure 5.1b) shows that the water quality does not comfort to class II (TN concentration = 0.5mg/L) in 2010 because all the player have no incentive to fully cooperate with others under the framework of the current policy. Therefore, the current game is only called a partially cooperative game because some players are still free riding.

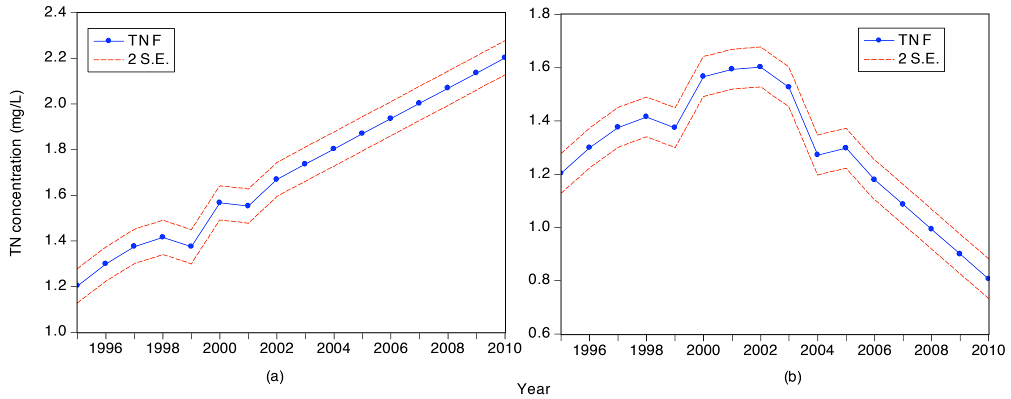

Figure 5.1: The results of annual average TN concentrations in (a) non-cooperative game scenario and (b) partially cooperative game scenario in Danjiangkou Reservoir

Therefore, the question is what policy is the suitable to make players fully cooperate with each other. In order to find the solution, it is necessary to analyze the prisoners' dilemma game. Suppose there are two players in order to simplify the issue. Figure 5.2 shows a 2-persons prisoners' dilemma game. Both players will be better off if they cooperate with each other. For every player in such situation, cooperation involves risk and uncertainty. Every player will risk the lowest payoff if others defect.

		Player 2: Cooperate	Player 2: Defect
Player 1	Cooperate	3 3	1 4
	Defect	4 1	2 * 2

Figure 5.2: A prisoners' dilemma game

The methods to solve the dilemma are to design a mechanism, which can change the rules and drive the players to reach collective rationality. The driving forces usually refer to something like laws, regulations, contracts and other binding agreement. However, these forces cannot work so well in the case of Danjiangkou Reservoir, because they could not make all the players fully cooperate. In contract with those legislation methods, economic methods such as tax, fine, compensation and so on, are also such kinds of driving forces. The game in figure 5.3 shows that the players have the incentive to choose cooperation and achieved the best payoff due to external driving forces. In this sense, the game is called *driving force game* (Figure 5.3). Solving the dilemma of TN reduction of Danjiangkou Reservior, the main conflicts between the reservoir catchment and the water receiving area should be analyzed. Reducing waste water and increasing water quality will impose cost to the reservoir catchment, but they mainly benefit water receiving area. In this case, all the players will have incentives to cooperate fully to reduce waste discharge if the benefits can also be transferred from the water receiving area to the reservoir catchment. However, the game model set up here cannot solve the problem of side payment. Therefore, another game model involving the water receiving area is very necessary to construct in the future.

Player 2

		Cooperate			Defect	
Player 1	Cooperate	4	*	4	1	3
	Defect	3		1	2	2

Figure 5.3: The driving force Game

6. Conclusions

Game theoretic models and notions can be applied to analyze the conflicts of reservoir water quality mangement. Some problems of water quality management in Danjiangkou Reservoir Catchment in China is analyzed and modeled as a game. The results show that non-cooperation will results in a prisoner's dilemma and exacerbate the water quality in Danjiangkou Reservoir. To solve the dilemma of TN reduction of Danjiangkou Reservoir, conflicts caused by water transfer should be analyzed and modeled. In the future, further game theoretical models will be worked out to demonstrate the usefulness and power of solving environmental conflicts.

Bibliography

Fundenberg, D., Tirole, J. (1996): Game Theory. MIT Press, Cambridge.

Pethig, R. (Ed.) (1992): Conflicts and Cooperation in Managing Environmental Resources. Berlin: springer-Verlag

Van der Veeren, R.J.H.M., Tol, R.S.J. (2003): Game theoretic analyses of nitrate emission reduction strategies in the Rhine river basin, Int. J. Global Environmental Issues, 3(1): 74-103.

Wang, L.Z. (et al) (2003): Water resource allocation: A cooperative game approach, Journal of Environmental Informatics, 2(1): 11-22.

Wei, S., Gnauck, A. (2007): Game theoretic approaches to model water conflicts on a river basin scale, in: Gnauck, A. (Hrsg): Modellierung und Simulation von Ökosystemen, Shaker Verlag: Aachen, pp. 22-40.