EnviroInfo 2007 (Warschau)
Environmental Informatics and Systems Research
 ISBN: 978-3-8322-6397-3

Pricing Financial Instruments Derivatives Inspired by Kyoto Protocol

Piotr Nowak[1], Maciej Romaniuk[2]

Abstract

In this paper the proposition of stochastic model which may be useful for pricing derivatives inspired by Kyoto Protocol is described. Based on neutral martingale method and Monte Carlo simulations the equation for price in case of European call option is provided.

1. Introduction

Since its adoption in the Kyoto Protocol text, emission trading has been seen as one of the primary tools for international co-operation to reduce emissions of greenhouse gases. This market-based instrument will be important mechanism for environmental protection (International Energy Agency 2001, United Nations Environment Programme 2002). Many countries will implement appropriate instruments for the first time, but the first tradable rights for pollution control were proposed much earlier, in 1986 (United Nations Environment Programme 2002).

Kyoto Protocol will probably stimulate two levels of markets for the described emissions trading instruments. The first one will be on world-wide scale, i.e. trading among countries. The second one is for individual countries markets. Both of these levels need the new financial mechanisms for decreasing risk, improving liquidity and rate of allowances turnover.

We know such instruments from the classical markets of options, futures and other derivatives for money and goods. Hence, there is a need to propose methodology appropriate for both the derivatives and very specific market of emission allowances.

The following paper is organized as follows. We describe general rules for emission allowances markets. We propose the stochastic process which may be suitable for these markets and find the mathematical formula for option price in example of rather standard option. We also discuss the possibility of simulations application for more complicated financial instruments. We present some conclusions and possible directions for future researches. Other important remarks on the discussed subject may be found in e.g. (Cason/Gangadharan, 1998; Devlin/Grafton, 1994; Ermoliev, 2000; Lemming, 2003; Montero, 1998; Stavins, 1995; Westkog, 1996).

2. Kyoto Protocol

International trade in greenhouse gas emissions is specifically provided for in the 1997 Kyoto Protocol (International Energy Agency 2001). The main aim for this treaty is to reduce emissions by a fixed percentage below what they were in 1990. These reductions are to be made during period 2008 – 2012. Because the necessary modification in industrial processes, upgrading factories and changing the course of

[1] Systems Research Institute, Polish Academy of Sciences, 01-447 Warsaw, ul. Newelska 6, Poland,
e-mail: pnowak@ibspan.waw.pl
[2] Systems Research Institute, Polish Academy of Sciences, 01-447 Warsaw, ul. Newelska 6, Poland,
e-mail: mroman@ibspan.waw.pl

national economies in many countries may be very costly, the Protocol proposed three innovative "market mechanisms", also including emission trading.

This emission trading will be based on the same general mechanisms as other financial markets. The main difference is that the tradable "good" will be greenhouse gas emissions permits specified in CO2 tons. The buyers of such permits will be companies (or countries for the worldwide market) in which the cost of reducing emissions is high. In contrary, the sellers will be entities for which this cost is lesser than the price of the permission, or the overall quantity of emission for them is lower than specified in Kyoto Protocol. Therefore, real functioning market will establish a market price for the emissions under some standard assumptions, like e.g. many enterprises involved in such trading.

The described above permissions may be treated as the primary financial market for some good. Easily seen, the bonds, the options, futures and forward contracts, as well as other financial derivatives may be established for such permission market. These instruments and the market of derivatives may be very useful especially for facilitating turnover, increasing flux liquidity and securing against overestimation of possible emission reductions and the risk connected with such overestimation error. Therefore, it is necessary to use adequate financial mathematics tools to solve problems arising from issuing or pricing of such instruments. These tools were highly developed in the past for the "standard" financial markets, like stock exchanges or futures markets (Buhlmann, 1996; Davis, 2001; Jacod/Shiryaev; 1987; Korn/Korn, 2001; Shiryaev/Kruzhilin; 1999, 2000).

But in the case of emissions trading, we should take into account some special settings, like the yearly reports of the reduction requirements fulfilment. Hence, before any problems could be solved using financial mathematics instruments, we are to develop appropriate model for permissions prices, which we propose in article. Obviously, this model should be adequate for permission market and should suitably forecast the future behaviour of the prices. Hence, we adopt a stochastic process which is generalization of the widely known Black-Scholes model with additional random jumps in the fixed time of the year. Such jump reflects the possibility of sudden change of the permission price course if the government or specific agenda announces the annual report about level of gas emissions for the given country and fulfilling the reductions from Kyoto Protocol.

Because of lack of the past data and relatively short history of the permission market, we may use the past experiences from similar markets. Exactly speaking, we use the idea of comparing CO2 market and other emissions allowances markets, e.g. for SO2. From the past data for these markets and general economic theory it could be seen that, the geometrical Brownian motion describing the trajectory of allowances prices is appropriate first approximation model. However, due to planned specification of the market derived from Kyoto Protocol, there is a need to slightly change such model in order to take into account periodical reports of the emission levels.

3. Option pricing

3.1 Mathematical preliminaries

Let $\left(\Omega, F, (F_t)_{t\in[0,T]}, P\right)$ be a probability space with filtration. Let $T < \infty$. A stochastic process $H = (H_t)_{t\in[0,T]}$ is cadlag, if its trajectories are right continuous with left limits. A cadlag stochastic process H is a semimartingale, if it admits a decomposition $H_t = A_t + M_t$, $t \in [0,T]$, where $A = (A_t)_{t\in[0,T]}$ is F_t-adapted cadlag process with finite variation and $M = (M_t)_{t\in[0,T]}$ is a local martingale.

We show an exposition of basic definitions and facts from (Shiryaev/Kruzhilin 1999/2000). Let $\varphi(x) = xI\{|x|\le 1\}$ be a truncation function. The process $H(\varphi)_t = H_t - \tilde{H}(\varphi)_t$, whe-

re $\breve{H}(\varphi)_t = \sum_{s \le t} [\Delta H_s - \varphi(\Delta H_s)]$, is a special semimartingale with canonical decomposition $H(\varphi) = X_0 + M(\varphi) + B(\varphi)$, i.e. $M(\varphi)$ is a local martingale and $B(\varphi)$ is a locally integrable, predictable process with finite variation.

Definition 1 Elements of triplet $T = (B, C, \nu)$ are called characteristics of H if
i) $B = B(\varphi)$ is a predictable process with finite variation;
ii) $C = < H^c >$, where H^c is the continuous martingale part of H;
iii) ν is predictable random measure on $B(T \times R)$ which is the compensator of the measure describing the jump structure of H, i.e. $\mu^H(dt, dx) = \sum_s I_{\{\Delta H_s \neq 0\}} \varepsilon(dt, dx)$.

Definition 2 A probability measure $\tilde{P}$ on (Ω, F) is locally absolutely continuous with respect to P ($\tilde{P} \overset{loc}{<<} P$), if for each $t \in [0, T]$ the probability measure $\tilde{P}_t = \tilde{P} \mid F_t$ is absolutely continuous with respect to $P_t = P \mid F_t$. If $\tilde{P} \overset{loc}{<<} P$ and $P \overset{loc}{<<} \tilde{P}$, then the measures are equivalent ($\tilde{P} \overset{loc}{\sim} P$).

Let $\tilde{P} \overset{loc}{\sim} P$. We define stochastic processes $Z_t = \frac{d\tilde{P}_t}{dP_t}$ and $M_t = \int_0^t \frac{dZ_s}{Z_{s-}}$, $t \in [0, T]$.

Let $\beta = \frac{d\langle Z^c, H^c \rangle}{d\langle H^c, H^c \rangle} \cdot \frac{I(Z_- > 0)}{Z_-}$ and $X = E^P_{\mu^H}\left(\frac{Z}{Z_-} I(Z_- > 0) \mid \tilde{P} \right)$, where $E^P_{\mu^H}$ is the expectation with respect to measure $M^P_{\mu^H}$ on $F \otimes B(T) \otimes B(R)$ defined by the equality $W * M^P_{\mu^H} = E(W * \mu^H)$ for all nonnegative measurable functions $W = W(\omega, t, x)$, and $<.,.>$ is the quadratic characteristic of processes.

The following theorem (Jacod/Shiryaev 1987) is called the Girsanov theorem for semimartingales.

Theorem. 3 Let $\tilde{P} \overset{loc}{<<} P$ and let Z, β and X be defined as above. Then $\tilde{B} = B + \beta C + \varphi(x)(X - 1) * \nu$, $\tilde{C} = C$ and $\tilde{\nu} = X\nu$ are the characteristics of H with respect to $\tilde{P}$.

3.2 Model of the underlying asset

The price of the underlying asset is the process $S = (S_t)_{t \in [0,T]}$ of the form $S_t = S_0 \exp(Y_t)$, where Y_t is F_t - adapted process defined by formula

$$Y_t = \mu t + \sigma W_t + \sum_{i=0}^{[t]} \xi_i , \tag{3.1}$$

W_t is a Brownian motion, $\sigma > 0$, $\mu \in R$, $\{\xi_i\}_{i=1}^{\infty}$ is the sequence of independent random variables with distribution $\rho(dx) = \left(\frac{\lambda_1}{2} e^{-\lambda_1 x} I_{\{x \geq 0\}} + \frac{\lambda_2}{2} e^{\lambda_2 x} I_{\{x<0\}}\right) dx$, $1 > \lambda_1, \lambda_2 > 0$ and [t] is the integer part of t.

3.3 Martingal method of option pricing

Let r be a constant risk-free rate and $\Xi_t = e^{-rt} S_t$ be the discounted process of the price of the underlying asset. Our aim is to find the measure $\widetilde{P}$, locally equivalent to P, such that Ξ_t is a martingale with respect to $\widetilde{P}$. The next step is to find the form of the process S_t with respect to the new measure $\widetilde{P}$. The price of the derivative with a payoff function f is given by formula:

$$C_t = \exp(-r(T-t)) E^{\widetilde{P}}(f(S) \mid F_t), \quad t \in [0,T] \tag{3.2}$$

We will apply the following result (Shiryaev/Kruzhilin 1999/2000). (We use the notation from Section 3.1.)

Theorem 4 Suppose that Z_t is a positive martingale with $dZ_t = Z_{t-} dM_t$, where M is given by

$$M_t = M_0 + \int_0^t \beta_s dH_s^C + \int_0^t \int_R W(.,s,x) d(\mu - \nu), \tag{3.3}$$

$W = X - 1 + \frac{X-a}{1-a} I(a<1)$, where $a = a_t(\omega) = \nu(\omega, \{t\} \times R)$, $X_t = \int_R X(\omega,s,x) \nu(\omega, \{t\} \times dx)$ and β and W satisfy corresponding integrability conditions (Jacod/Shiryaev 1987). Moreover, assume that $E \mid Z_t \mid = 1$. Then in the cases where $\nu(\omega, \{t\} \times R) \in \{0,1\}$, condition

$$K_t + \int_0^t \beta_s d\langle H^C \rangle_s + \int_0^t \int_R (X-1)(e^x - 1) d\nu = 0, \quad t \leq T, \tag{3.4}$$

where $K_t = B_t + \frac{1}{2} \langle H^C \rangle_t + \int_0^t \int_R (e^x - 1 - \varphi(x)) d\nu$, implies the existence of a measure $\widetilde{P}_T$, constructed as above, equivalent to the measure P_T, for which Ξ is a local martingale.

For $\gamma > \frac{1}{2}$ we define $\tilde{\rho}(dx) = \left(\frac{\gamma + \frac{1}{2}}{2} e^{-\left(\gamma + \frac{1}{2}\right)x} I_{\{x \geq 0\}} + \frac{\gamma - \frac{1}{2}}{2} e^{\left(\gamma - \frac{1}{2}\right)x} I_{\{x<0\}} \right) dx$. (3.5)

Theorem 5 For European call option with payoff function $f(x) = (x-K)_+$, $K > 0$,

$$C_0 = \exp(-rT) \int_{\ln K - \left(r - \frac{\sigma^2}{2}\right)T}^{\infty} \left(S_0 e^{\left(r - \frac{\sigma^2}{2}\right)T + x} - K \right) dG^{[T]}(x), \tag{3.6}$$

where $G^{[T]}(x) = G * \overbrace{F * \ldots * F}^{[T]}(x)$ and G and F are cumulative distribution functions of normal distribution $N\left(0, \sigma\sqrt{T}\right)$ and $\tilde{\rho}$, respectively. $G^{[T]}$ has density function $g^{[T]}$ and the above formula can be written as follows:

$$C_0 = e^{-\frac{1}{2}\sigma^2 T} \int_{\ln \frac{K}{S_0} - \left(r - \frac{\sigma^2}{2}\right)T}^{\infty} e^x g^{[T]}(x) dx - e^{-rT} K \left(1 - G^{[T]}\left(\ln \frac{K}{S_0} - \left(r - \frac{\sigma^2}{2} \right) T \right) \right).$$

Proof. The characteristics B and C of the process $H_t = Y_t - rt$ have the form $B_t = (\mu - r)t + [t] \int_R \varphi(x) \rho(dx)$ and $C_t = \sigma^2 t$. Since H is a process with independent increments, formula (3.3) shows the decomposition of M. All the assumptions of Theorem 4 are satisfied. Since

$$K_t = (\mu - r)t + [t] \int_R \varphi(x) \rho(dx) + \frac{1}{2}\sigma^2 t + [t] \int_R \left(e^x - 1 - \varphi(x) \right) \rho(dx),$$

equation (3.4) is equivalent to

$$\left(\mu - r + \frac{1}{2}\sigma^2 \right) t + \sigma^2 \int_0^t \beta_s ds + [t] \left(\int_R X(\omega, 1, x) \left(e^x - 1 \right) \rho(dx) \right) = 0, \quad t \leq T. \tag{3.7}$$

Sum $I_1 = \left(\mu - r + \frac{1}{2}\sigma^2 \right) t + \sigma^2 \int_0^t \beta_s ds$ is continuous as the function of t and $I_2 = [t] \int_R X(\omega, 1, x) \left(e^x - 1 \right) \rho(dx)$ has jumps for $t = 1, 2, \ldots$. We obtain the following solution of (3.7):

$\beta = -\frac{(\mu - r)}{\sigma^2} - \frac{1}{2}$, $X(\omega,t,x) = I_{\{t\in N, x\geq 0\}} e^{\left(-\frac{1}{2}-\gamma+\lambda_1\right)x} / R_1 + I_{\{t\in N, x<0\}} e^{\left(-\frac{1}{2}+\gamma+\lambda_2\right)x} / R_2 + I_{\{t\notin N\}}$

for $R_1 = 2Ee^{\left(-\frac{1}{2}-\gamma+\lambda_1\right)\xi_1} I_{\{\xi_1\geq 0\}}$ and $R_2 = 2Ee^{\left(-\frac{1}{2}+\gamma+\lambda_2\right)\xi_1} I_{\{\xi_1<0\}}$. Applying Theorem 3 for characteristics of H with respect to $\tilde{P}_T$, we obtain the characteristics of Y with respect to $\tilde{P}_T$ of the form $\tilde{B}_t^Y = \left(r - \frac{1}{2}\sigma^2\right)t + [t]\int_R \varphi(x) X(\omega,1,x)\rho(dx)$, $\tilde{C}_t^Y = \sigma^2 t$, $\tilde{\nu}^Y(\omega,(0,t]\times dx) = [t] X(\omega,1,x)\rho(dx)$. Therefore the form of process Y with respect to $\tilde{P}_T$ is described by formula $Y_t = \left(r - \frac{1}{2}\sigma^2\right)t + \sigma\tilde{W}_t + \sum_{i=0}^{[t]} \tilde{\xi}_i$, where $\tilde{W}_t$ is a Brownian motion, $\{\tilde{\xi}_i\}_{i=1}^{\infty}$ are independent random variables with distribution $\tilde{\rho}(dx) = X(\omega,1,x)\rho(dx)$. If $F(x) = \tilde{\rho}((-\infty,x])$ is the cumulative distribution function of $\tilde{\xi}_i$ and $F^{[T]}(x) = \overbrace{F * \ldots * F}^{[T]}(x)$, then $G^{[T]}(x) = G * \overbrace{F * \ldots * F}^{[T]}(x)$ is the cumulative distribution function of $Y_T - \left(r - \frac{1}{2}\sigma^2\right)T$. Therefore $Y_T - \left(r - \frac{1}{2}\sigma^2\right)T$ has the density. From (3.2) it follows that

$$C_0 = \exp(-rT)E^{\tilde{P}} f(S_T) = \exp(-rT)\left[E^{\tilde{P}} S_T I_{S_T > K} - K\tilde{P}(S_T > K)\right] =$$

$$e^{-\frac{1}{2}\sigma^2 T} \int_{\ln\frac{K}{S_0} - \left(r - \frac{\sigma^2}{2}\right)T}^{\infty} e^x g^{[T]}(x)dx - e^{-rT} K\left(1 - G^{[T]}\left(\ln\frac{K}{S_0} - \left(r - \frac{\sigma^2}{2}\right)T\right)\right). \tag{3.8}$$

3.4 Monte Carlo simulations

Easily seen, the stochastic process which describes the price of the allowance is rather complicated for direct calculations, especially because of the formula for density (3.5). Even for the simplest case of European call function, the price formula (3.8) may not be analytically tractable.

Hence, there is a need to use numerical or simulations methods in order to find computable formula. These methods could be also helpful for pricing other, more complex derivatives based on the proposed process (3.1) and its martingale modification obtained in Sec. 3.3. We use Monte Carlo simulations, which are very flexible and efficient even for large portfolios of financial instruments. This approach may be especially suitable for entities which calculate possible costs and revenues of different strategies for emissions reduction based on various instruments, e.g. not only financial, but also on alternation of industrial processes, changing scope of production, etc. In such case, the simulations of possible scenarios are natural way to incorporate these different mechanisms and to provide basis of decision support system.

In Monte Carlo simulation we are to find the price for the derivative:

$$C_0 = \exp(-rT)E(FV(f(S_t))) \ , \tag{3.9}$$

which is the discounted expected value for future cash flows of payment function *f(.)* based on underlying asset trajectory S_t modelled by the given stochastic process. Therefore, we have to simulate *n* trajectories $S_t^{(1)}, S_t^{(2)}, \ldots, S_t^{(n)}$, where *n* is called *number of trajectories* and then calculate the classical estimator based on average. For European call function, we have $C_0 = \exp(-rT)\frac{1}{n}\sum_{i=1}^{n}\left(S_T^{(i)} - K\right)_+$, where $S_T^{(i)}$ is price for the *i*-th underlying asset trajectory at time moment *T*.

If we divide the time interval *[0,T]* into *m* equal steps $t_0 = 0, t_1, \ldots, t_m = T$, then we could discretize the trajectory $S_t^{(i)}$ into segments $S_{t_0}^{(i)}, S_{t_1}^{(i)}, \ldots, S_{t_m}^{(i)}$, where *m* is called number of steps. From martingale modification of process (3.1) we have

$$S_{t_{j+1}}^{(i)} = S_{t_j}^{(i)} \exp\left(\left(r - \frac{1}{2}\sigma^2\right)\Delta t_j + \sigma N_{i,j}\Delta t_j + \lfloor \Delta t_j \rfloor P_{i,j}\right) \tag{3.10}$$

where $\Delta t_j = t_{j+1} - t_j$, $N_{i,j}$ are *iid* (independent, identically distributed) random variables from *N(0;1)* (standard normal distribution), $P_{i,j}$ are *iid* random variables with distribution given by (3.5), and $\lfloor \Delta t_j \rfloor$ is number of "jump events" in interval Δt_j, i.e. $\lfloor \Delta t_j \rfloor := [t][\Delta t_j]$. Therefore, from (3.10), which is similar to Euler's scheme known for Black--Scholes model, we could simulate necessary steps $S_{t_0}^{(i)}, S_{t_1}^{(i)}, \ldots, S_{t_m}^{(i)}$ for

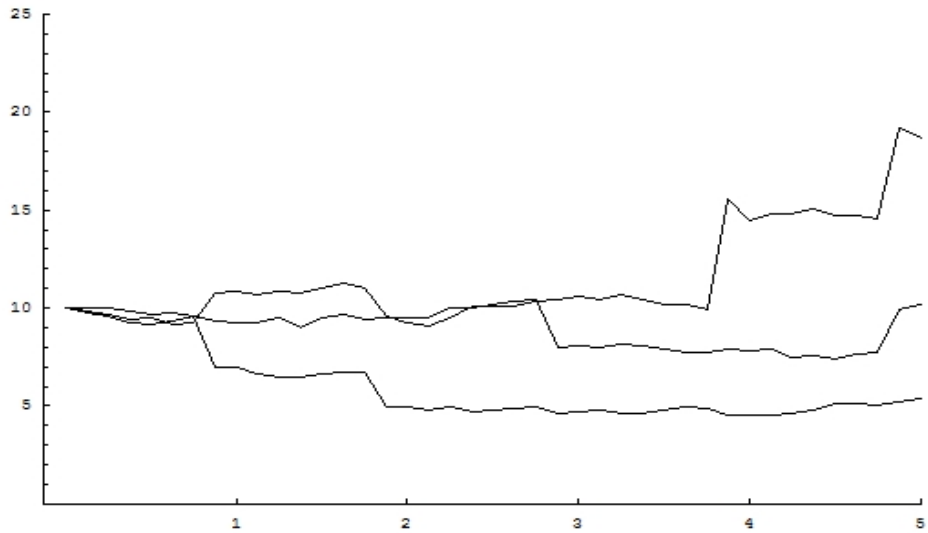

Figure 1: Example of a few paths of the process generated by Monte Carlo simulations

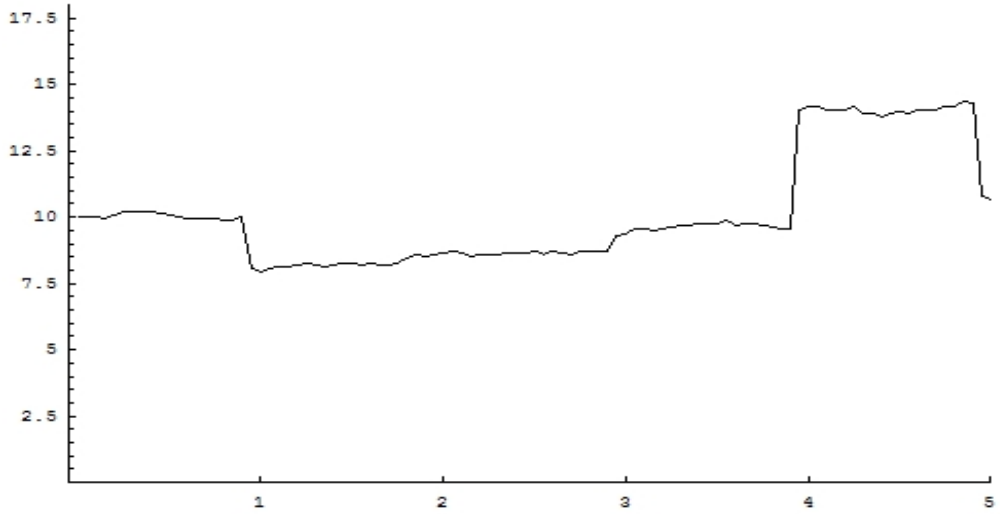

Figure 2: More detailed trajectory of the process generated by Monte Carlo methods

any trajectory $S_t^{(i)}$. Applying the data from these trajectories to (3.9), we could find the estimator of the price for the given kind of derivative.

We have prepared appropriate simulations to illustrate our considerations. The example of a few price trajectories generated by Monte Carlo simulations is shown at Figure 1. As we could see, these trajectories are similar to the one obtained from classical Black-Scholes model (i.e. modelled by geometrical Wiener process), but with occasional "shocks" added by $P_{i,j}$ part in (3.10). This property may be better seen at Figure 2, where the number of steps was greater and shocks had greater volatility.

We have conducted 10 000 simulations using parameters m=5, r=0.04, σ=0.2,γ=0.6, λ1=0.9, λ2=0.1, S0=10, T=5, K=12. The price for European call function was 3.96595. Because the central limit theorem could be applied for such setting, also the 95% confidence interval for price was calculated. For the mentioned parameters it was equal to [3.81049, 4.12141]. Of course, if we use more simulations, more accurate price may be found.

We have also found that the prices are rather sensitive to the "jump parameters" (i.e. λ1 and λ2). The mentioned effect may be seen in Table 1, where the prices for various values of λ2 were calculated and all the other parameters were constant.

Table 1:
Effect of various "shock parameters" for option price

Value of λ_2	Option price	Lower limit of conf. int.	Upper limit of conf. int.
0.1	3.96595	3.81049	4.12141
0.2	3.23153	3.06852	3.39454
0.3	2.78831	2.6415	2.93511
0.4	2.48164	2.3444	2.61888

4. Conclusions

In this paper we discuss some general issues concerning emissions allowances market arising from Kyoto Protocol. We point out that introduction of such financial instruments, like options, futures and forward, known as derivatives, will help in the decreasing risk and improving liquidity of the evolving emissions market. These derivatives based on emissions allowances are similar to the analogous instruments for classical financial markets.

However, there are some differences, and one of them is the model for underlying asset trajectory. We propose such model based on generalization of classical Black-Scholes model. We also discuss the possibility of simulations application for the finding of price option estimator. We present some calculations based on the simulations for European call option.

The possible future directions of researches include comparison between the predictions based on our model and the real market, estimation of parameters required by the described stochastic process and the generalization and fitting of this model.

Bibliography

Buhlmann, H., Delbaen, F., Embrechts, P., Shiryaev, A.N. (1996): No-arbitrage, change of measure and conditional Esscher transforms, CWI Quarterly, 9(4)

Cason, T. N., Gangadharan, L. (1998): An experimental study of electronic bulletin board trading for emission permits, Journal of Regulatory Economics, 14(1): 55-73

Davis, M. (2001): Mathematics of financial markets, in: Engquist B., Schmid W.: Mathematics Unlimited – 2001 & Beyond, Springer Berlin

Devlin, R. A., Grafton, R. Q. (1994): Tradable Permits, Missing Markets and Technology, Environmental and Resource Economics, 4: 171-186

Ermoliev, Y., Michalevich, M. (et al.) (2000): Markets for Tradable Emissions and Ambient Permits: A Dynamic Approach, Environmental and Resource Economics, 15(1): 3-56

International Energy Agency (2001): International Emission Trading. From Concept to Reality

Jacod J., Shiryaev, A.N. (1987): Limit theorems for stochastic processes, Springer Verlag

Korn, R., Korn, E. (2001): Option Pricing and Portfolio Optimization, American Mathematical Society

Leming, J. (2003): Financial Risks for Green Electricity Investors and Producers in a Tradable Green Certificate Market, Energy Policy, 31(1): 21-32

Montero, J.-P. (1998): Marketable Pollution Permits with Uncertainty and Transactions Costs, Resource and Energy Economics, 20 (1): 27-50

Shiryaev, A. N., Kruzhilin, N. (1999/2000): Essential of Stochastic Finance, World Scientific Publishing Co. Pte. Ltd.

Stavins, R. N. (1995): Transaction Costs and Tradable Permits, Journal of Environmental Economics and Management, 29(2): 133-148

United Nations Environment Programme, Division of Technology, Industry and Economics (2002): A guide to Emissions Trading

Westkog, H. (1996): Market Power in a System of Tradable CO2 Quotas, The Energy Journal, 17: 85-103