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A Method for High Precision Coordinate Transformation in Environmental GIS Applications

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Abstract

Development of a method for coordinate system transformations, which can be used within GIS applications for environmental information management, is described. Transformation of geographic (in *degrees*) into geodetic (in *m*) coordinates was investigated using the old Yugoslav coordinate system (still in use in the newly established states of Slovenia, Croatia, Bosnia and Herzegovina, Serbia, Montenegro and Macedonia) as source data, with precision of *cm*. To simplify the transformation procedure, the general polynomial coordinate transformation was used, organized in several steps. First, an equation of the third order for interpolation of northing coordinates along a central meridian, as a function of latitude, was developed using a central difference scheme. Using the results obtained in such a way equations of second and fourth orders were developed for calculation of the increase in northing and shifts of easting from false easting, as a function of longitude (for a distance of 2° from the central meridian) along the corresponding parallel. Interpolation by 1° resulted in errors up to 5 cm, while interpolation by 30' resulted in errors below 1 cm, which corresponds to the precision of original source data. Interpolation by 15' or less cannot further increase the accuracy. Quadratic terms (about 485m/°) are dominant in equations for the increase in northing and linear terms (about 80000m/°) are dominant in those for the shift in easting, while other terms are in the range of centimetres or less.

1. Introduction

Environmental information has been generated and used at different locations in the course of time and, therefore, temporal and spatial presentation of environmental information is a prerequisite for its efficient management (Pecar-Ilic/Ruzic, 2001). The European Union recognized these needs during the last decade and made a significant investment into research and technology development for the use of telematic applications to the problems of environmental management and decision making (EC 1999). For this purpose, there is a need to develop an integrated information system for temporal and spatial presentation of complex data, using the object-oriented (OO) approach (Pecar-Ilic, 2001). Based on the performed WEB-GIS-DBMS integration, it is possible to analyze all the necessary types of information, so that presentations such as maps, reports, time series diagrams and others could be obtained in due time. The corresponding applications using interactive digital maps can be created with the aid of advanced GIS tools such as: ESRI ArcInfo®, Autodesk Map® 3D, MapInfo Professional®, Intergraph GeoMedia®, etc.

1.1 Methods for coordinate system transformation

It has been recognized that it is frequently required to convert coordinates derived in one geographic coordinate system to values expressed in another. In order to harmonize geo-related information for international, regional and bilateral projects, a conversion is needed between national and WGS 84 geographic coordinates into the national geodetic system and vice versa. In the early days of satellite surveying when relationships between geodetic datums were not well defined, it was usual to apply merely a 3-parameter dX, dY, dZ shift to the XYZ coordinate set in one datum to derive those in the other datum. Improved accuracy can be obtained by applying a 7-parameter transformation, which in addition to dX, dY and dZ shifts to the translation vector include also three rotations, R_x, R_Y and R_Z, of the position vector, and finally the

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scale correction m to the position vector in the source coordinate system in order to obtain the correct scale in the target coordinate system (POSC, 2000).

$$\lfloor X_T, Y_T, Z_T \rfloor = m * [1 - R_Z \; R_Y, R_Z \; 1 \; R_Y, - R_Y \; R_X \; 1 * X_S, Y_S, Z_S + dX, dY, dZ \tag{1.1.1}$$

where X_T, Y_T and Z_T, as well as X_S, Y_S, Z_S, are the coordinates of a point in the target and source coordinate systems, respectively, and m is the scale correction expressed in parts per million.

1.2 Problems in using commercial softwares

During the production of the Croatian electronic river navigation charts, we used older versions of Autodesk AutoCAD Map® software tools based on these methods, which did not transform accurately Croatian national geodetic data into national and WGS 84 geographic coordinates. We therefore designed a correction procedure that could be efficiently used to obtain satisfactory results (Ruzic/Pecar-Ilic/Eres, 2004).

The correction applied includes both the translation for latitude and the first order correction for latitude and longitude (equivalent to the known affine parametric transformation). The remaining error ranged from 0 to 11 cm in latitude and from 0 to 16 cm in longitude. The correction procedure is equivalent to the known affine parametric transformation, which can be described in the following way (POSC, 2000; OGP, 2006):

$$[X_T, Y_T] = [X_0, Y_0] + [A_1 \; A_2, B_1 \; B_2] * [X_S, Y_S] \tag{1.2.1}$$

where X_0, Y_0, A_1, A_2, B_1 and B_2 represent the corresponding coefficients to be determined. Further correction is possible by using higher order terms, which would produce errors on a scale of centimetres. New versions of the Autodesk software, now available on the market, have apparently corrected some of these errors. However, we discovered that they (the old one as well as the new versions) still do not transform accurately Croatian national geodetic data into the WGS 84 system. We applied a similar procedure to resolve these difficulties in order to produce accurate WGS 84 compatible geographic coordinates for Croatia. Affine transformation is analogous to the general polynomial transformation formulas and can be considered as a first order general polynomial transformation (OGC,2001; OGP, 2006; POSC, 2000).

The purpose of this paper is to examine the use of general polynomial transformations for conversion of geographic to geodetic coordinates. If efficient and accurate enough, it can be used to test and correct national geo-related data.

2. Development of the method for coordinate transformations

Alternatively, instead of using 3 or 7 parameters coordinate transformations, a polynomial expression with listed coefficients for both latitude and longitude may be used as the transformation method. Polynomial transformations between two coordinate systems are typically applied in cases where one or both of the coordinate systems exhibit lack of homogeneity in orientation and scale. The small distortions are then approximated by polynomial functions in latitude and longitude or in easting and northing (OGP, 2006). Depending on the degree of variability in the distortions, approximation may be carried out using polynomials of degree 2, 3, or higher.

Our intention is to develop a special procedure to reduce the number of terms needed for the coordinate transformation by using a gradual interpolation. For example, if we examine the interpolation of northing coordinates X along a central meridian (CM), then only a dependence on latitude should be considered:

$$m_T dX = A_0 + A_1 U + A_2 U^2 + A_3 U^3 + A_4 U^4 + \ldots \tag{2.0.1}$$

The use of a central difference scheme can significantly simplify the procedure for such interpolation. In addition, we examined the interpolation of northing (*X*) and easting (*Y*) along a parallel where dependence on longitude should only be considered:

$$m_T dX = A_0 + A_1 V + A_2 V^2 + A_3 V^3 + A_4 V^4 + \ldots \tag{2.0.2}$$

$$m_T dY = B_0 + B_1 V + B_2 V^2 + B_3 V^3 + B_4 V^4 + \ldots \tag{2.0.3}$$

where: $U = m_S(\varphi_S - \varphi_{S0})$ and $V = m_S(\lambda_S - \lambda_{S0})$ – scaled offsets for latitude and longitude; *T*, *S* – target and source data, respectively; A_0 – obtained by interpolation along the central meridian, using eqn. (2.0.1) and B_0 – equivalent to the false easting.

2.1 Interpolation along a central meridian

We could reduce the number of terms by using gradual interpolation. For example, if we examine the interpolation of northing coordinates along a central meridian, then only the dependence of northing on latitude should be considered (see eqn. 2.0.1).

Table 1:
Central difference for 4th order interpolation

Source data	**1st difference**	**2nd diff./2**	**3rd diff./3**	**4th diff./4**
$A_0 \pm 3A_1 + 9A_2 \pm 27A_3 + 81A_4 \pm \ldots$				
	$A_1 \pm 5A_2 + 19A_3 \pm 65A_4 + \ldots$			
$A_0 \pm 2A_1 + 4A_2 \pm 8A_3 + 16A_4 \pm \ldots$		$A_2 \pm 6A_3 + 25A_4 \pm \ldots$		
	$A_1 \pm 3A_2 + 7A_3 \pm 15A_4 + \ldots$		$A_3 \pm 6A_4 + \ldots$	
$A_0 \pm A_1 + A_2 \pm A_3 + A_4 \pm \ldots$		$A_2 \pm 3A_3 + 7A_4 \pm \ldots$		$A_4 \pm \ldots$
	$A_1 \pm A_2 + A_3 \pm A_4 + \ldots$		$A_3 \pm 2A_4 + \ldots$	
A_0		$A_2 + A_4 + \ldots$		$A_4 + \ldots$
$\mathbf{A_0 = \Sigma/7 - 4A_2 - 29A_4 \ldots}$	$\mathbf{A_1 = \Sigma/6 - 9A_3 + \ldots}$	$\mathbf{A_2 = \Sigma/5 - 13A_4 + \ldots}$	$\mathbf{A_3 = \Sigma/4 + \ldots}$	$\mathbf{A_4 = \Sigma/3 \pm \ldots}$

In Table 1, within the equations (shown by shading) developed for calculation of coefficients A_i, symbols Σ represent the sum of terms in columns. Use of the central difference scheme can significantly simplify the procedure for interpolation. Here, the free coefficient A_0 is used only to check the accuracy of interpolation.

2.2 Interpolation along a parallel

If we examine the interpolation of northing (*X*) and easting (*Y*) along a parallel, then their dependence only on longitude should be considered (see eqns. 2.0.2 and 2.0.3). Use of the difference scheme with the origin from the central meridian, shown in Table 2, can again significantly simplify the determination procedure for the corresponding A_i and B_i coefficients.

Table 2:
Additional difference scheme for 4th order interpolation

Source data	1st difference	2nd diff./2	3rd diff./3	4th diff./4
$A_0+4A_1+16A_2+64A_3+256A_4+\ldots$				
	$A_1+7A_2+37A_3+175A_4+\ldots$			
$A_0+3A_1+9A_2+27A_3+81A_4+\ldots$		$A_2+9A_3+55A_4+\ldots$		
	$A_1+5A_2+19A_3+65A_4+\ldots$		$A_3+10A_4+\ldots$	
$A_0+2A_1+4A_2+8A_3+16A_4+\ldots$		$A_2+6A_3+25A_4+\ldots$		A_4
	$A_1+3A_2+7A_3+15A_4+\ldots$		$A_3+6A_4+\ldots$	
$A_0+A_1+A_2+A_3+A_4+\ldots$		$A_2+3A_3+7A_4+\ldots$		
	$A_1+A_2+A_3+A_4+\ldots$			
A_0				
$A_0=(\Sigma-354A_4)/5-2A_2-A_3-\ldots$	**$A_1=\Sigma/4-4A_2-16A_3-64A_4-\ldots$**	**$A_2=\Sigma/3-6A_3-29A_4-\ldots$**	**$A_3=\Sigma/2-8A_4-\ldots$**	**A_4**

Within the equations (shown by shading in Table 2) developed for calculation of coefficients A_i, symbols Σ represent the sum of terms in columns. A completely equivalent difference scheme can be used for determination of coefficients B_i from eqn. (2.0.3). Here again the free coefficients A_0 and B_0 are used only to check the accuracy of interpolation.

3. RESULTS

We decided to test the feasibility of the polynomial coordinate transformation for the calculation of national geodetic coordinates from geographic coordinates of the old Yugoslav system, still used by all the newly established states (Slovenia, Croatia, Bosnia & Herzegovina, Serbia, Montenegro and Macedonia).

Table 3:
A list of geographic and X (northing) geodetic coordinates as a function of longitude
for the central meridian

Coordinates to be interpolated						
Geographic coordinates						
41°	42°	43°	44°	45°	46°	47°
X-Geodetic						
4539662.99	4650703.35	4761763.08	4872842.22	4983940.83	5095058.90	5206196.46
1st Difference						
111040.36	111059.73	111079.14	111098.61	111118.07	111137.56	
2nd Difference/2						
9.685	9.705	9.735	9.730	9.745		
3rd Difference/3						
0.0066	0.0100	-0.0025	0.0050			

For this purpose, we used the corresponding Catalogue of Gauss-Krueger Coordinates, produced by the former Yugoslav Federal Directorate for Geodesy, which are given with precision of *cm* (FDG, 1964). First, we interpolated data for the central meridian using the simple cubic function of latitude described by eqn. (2.0.1) for degrees from 41^o to 47^o, with their 1st, 2nd and 3rd order differences (see Table 3).
After that, the same Catalogue of Gauss-Krueger coordinates (FDG, 1964) was used to calculate national geodetic coordinates at a distance from the central meridian. However, this time for the parallel 45o, we interpolated *X* (northing) data using eqn. (2.0.2) and *Y* (easting) data using eqn. (2.0.3). A list of source coordinates (*X* and *Y*) for each 30' of longitude (30', 1o, 1o30' and 2o) was used, with their 1st, 2nd, and 3rd differences (see Table 4).

Table 4:
A list of geographic, X (northing) and Y (easting) geodetic coordinates as a function of longitude for the parallel 45^o

Coordinates to be interpolated				
Geographic coordinates				
Central Meridian CM	CM + 30'	CM + 1^o	CM + 1^{o}30'	CM + 2^o
X-Geodetic				
4983940.83	4984062.43	4984427.28	4985035.42	4985886.94
1st Difference				
121.60	364.85	608.14	851.52	
2nd Difference/2				
121.625	121.645	121.690		
3rd Difference/3				
0.0066	0.0150			
Y-Geodetic				
500000.00	539414.71	578829.42	618244.14	657658.87
1st Difference				
39414.71	39414.71	39414.72	39414.73	
2nd Difference/2				
0.0	0.005	0.005		
3rd Difference/3				
0.0016	0.0			

The results of these interpolations are presented in Table 5. For the parallel 45^o, it is evident that the quadratic term for northing and the linear term for easting are dominant.

Table 5:
Coefficients of the national cartographic system from eqns. (2.0.1), (2.0.2) and (2.0.3)

Coefficients from eqn. (2.0.1) for the central meridian in *m*				
A_0=4872842.239	A_1=111088.869	A_2=9.72	A_3=0.00479167	–
Coefficients from eqns. (2.0.2) and (2.0.3) for the parallel 45° in *m*				
A_0=4983940.83	A_1=-0.04833	A_2=486.512	A_3=-0.04666	A_4=0.03333
B_0=500000.00	B_1=78829.4316	B_2=-0.0383	B_3=0.03333	B_4=-0.00666

Associated errors are shown in Table 6. Errors for the central meridian range between -0.68 cm and 1.86 cm and they correspond to the precision of the original geodetic coordinates used. If we use for interpolation a list of *X* coordinates for each 30 minutes, errors range from -0.51 cm to 0.98 cm.

Errors for the parallel 45° range from -0.3 cm to 0.61 cm. Interpolation by 1° of longitude (1° and 2°) with the 1st and 2nd differences produces larger errors, from 3.875 cm to 5.125 cm for northing and relatively small errors for easting (from -0.375 cm to 0.125 cm) coordinates. If we use only a quadratic term for northing, errors range from 0 to 9.69 cm; when we use only a linear term for easting, errors range only from 0 to 1.5 cm.

Table 6:
Errors in calculation of northing and easting coordinates in *cm*, using eqns. (2.0.1), (2.0.2), (2.0.3) and coefficients A_i and B_i from Table 5

Interpolation method	**Northing, CM**	**Northing, parallel 45°**	**Easting, parallel 45°**
single term	–	0 to 9.69 (only A_2)	0 to 1.5 (only B_1)
by degrees	-0.68 to 1.86	3.875 to 5.125	-0.375 to 0.125
by 30 minutes	-0.51 to 0.98	-0.3 to 0.4	-0.14 to 0.61

Use of a list of X coordinates by 15 minutes or less for both cases could not increase the accuracy of interpolation.

4. COMPLETE GENERATION OF GEODETIC COORDINATES

Interpolation coefficients for parallels other than 45° will be different and complete geodetic coordinates could be generated with additional coefficients for mixed terms. Then, the corresponding A_i and B_i terms (where $i = 0, 1, 2, ...$) from eqns. (2.0.2) and (2.0.3) are not constants any more, but functions of *U* coordinate. These functions can be presented in the following way:

$$A_i = a_{0i} + a_{1i}U + a_{2i}U^2 + ... \quad \text{and} \quad B_i = b_{0i} + b_{1i}U + b_{2i}U^2 + ... \tag{4.0.1}$$

Coefficient a_{00} coincides with A0 obtained from the eqn. (2.0.1), while coefficient b_{00} coincides with the false easting B0. From the source data of the old Yugoslav cartographic system, the primary coefficients aij and bij are obtained using mainly quadratic interpolation. These primary coefficients should be determined once for the national cartographic system. Then from different input geographic coordinates, using eqns. (2.0.2) and (2.0.3), one can obtain the corresponding output geodetic coordinates. The complete coordinate transformation procedure is represented in Figure 1. In our case, the coefficients obtained are shown in Table 7 and the associated errors are presented in Table 8.

Table 7:
Coefficients from eqn. (4.0.1) for the national cartographic system

Primary coefficients for calculation of secondary coefficients Ai in metres			
a_{01}=-0.1900	a_{11}=0.0233	a_{21}=0.0100	–
a_{02}=486.3054	a_{12}=0.6033	a_{22}=-0.3010	–
Primary coefficients for calculation of secondary coefficients B_i in metres			
b_{01}=80188.19	b_{11}=-1346.57	b_{21}=-12.294	b_{31}=0.0675
b_{02}=0.4371	b_{12}=-0.4292	b_{22}=0.0100	–

It is evident again that the quadratic term is dominant in the equation for northing and the linear term is dominant in the equation for easting.

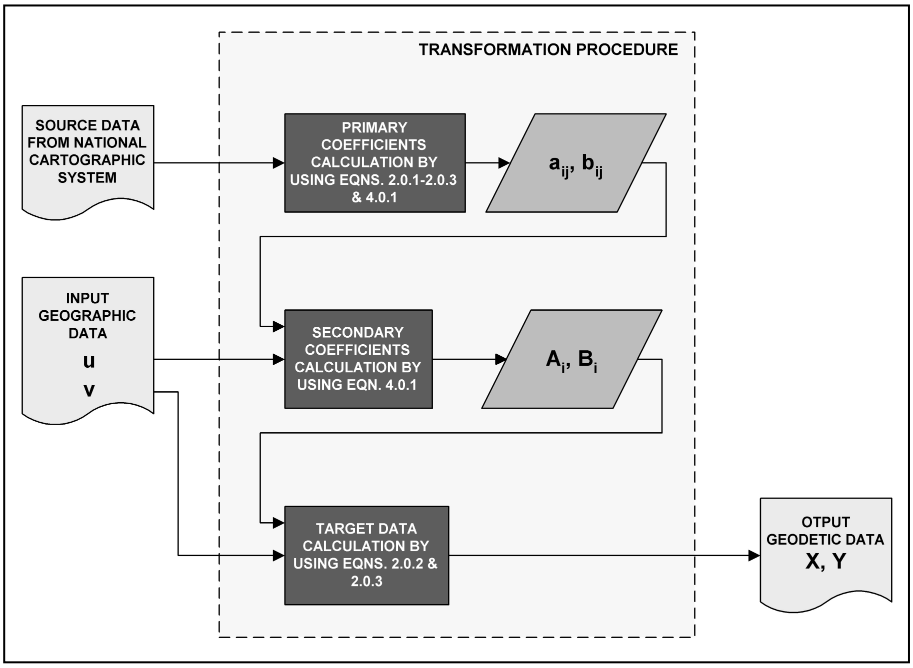

Figure 1: Procedure for coordinate transformation

The resulting errors in generating northing coordinates range from -3.0 to 5.2 cm for the linear term and from -2.7 to 1.6 cm for the quadratic term. The resulting errors in generating easting coordinates range from -1.9 to 2.2 cm for the linear term and from -1.4 to 0.9 cm for the quadratic term.

Table 8:
Errors in calculation of secondary coefficients in *cm*, using eqn. (4.0.1)

Interpolation method	A_1	A_2	B_1	B_2
single term	–	-35 to 8 (only a_{02})	–	–
by degrees	-3.0 to 5.2	-2.7 to 1.6	-1.9 to 2.2 (if a_{31}>0)	-1.4 to 0.9
by 30 minutes	< 1	< 1	< 1	< 1

Without the third order term b_{31}, the error for function $B_1(U)$ would be very high (-182 to 182 cm). Use of higher order terms can reduce these errors to values less than 1 cm. However, the use of only the quadratic term a_{02} for northing or only the linear term for easting would again produce very large errors between -35 cm and 8 cm. Use of higher order primary coefficients can reduce all errors for both northing and easting to values less than 1 cm.

5. CONCLUSIONS

Although most commercial software packages for geographic coordinate transformation offer possibilities to use different types of methods (such as Helmert 7-parameters method), high accuracy can be achieved only for a limited area. Discrepancies of the order of less than 60 cm are reported for networks of 100 km in size, while discrepancies less than 20 cm are reported for networks of 10 km in size (Mitsakaki, 2004). For example, in the case of the Croatian State Coordinate System (HDKS-2001), which is based on the Gauss-Krueger coordinates and Bessel 1841 ellipsoid, transformation parameters for 20 different districts and 20 cities have been determined (CSDG, 2003). They correspond to the area within the radius of less than 30'. It is therefore always necessary to check the accuracy of software packages used. The polynomial coordinate transformation method described in this paper can significantly simplify the whole process and at the same time accuracies better than 1 cm can be obtained. Using primary coefficients determined in this paper, eqns. (4.0.1) could be incorporated within the commercial GIS software packages (for example Autodesk Map® 3D) in order to enable automatic coordinate transformation. Similar procedures can be developed for any other national data if accurate source data are available. The procedure developed can be used for a wide range of coordinates with accuracies corresponding to the precision of the original source data.

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