

UPIOM Cube: A New Tool for Visualization of Inter-industry Flow of Various Types of Material with Its Application to Car Production

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Abstract

This paper proposes “UPIOM Cube”, a new tool for graphical visualization of the economy-wide inter-sector flows of an arbitrary number of materials. The tool is based on the hybrid input-output methods developed by ourselves in earlier work. The quality of metals recovered from an end-of-life product is degraded if various metals are mixed in the end-of-life process. It is thus indispensable to consider different materials simultaneously in a material flow analysis for sustainable material management. In order to demonstrate the utility of UPIOM Cube, we apply the tool to flows of iron and copper associated with passenger car production in Japan.

1. Introduction

Diverse metal species are mainly used in combination with each other in manufacturing. The quality of metals recovered from an end-of-life product would become degraded if various metals are mixed in the end-of-life process because some of the impurities in metal-melting processes occur as troublesome tramp elements (Nakajima et al. 2010). Therefore, it is indispensable to take different materials into account simultaneously in a material flow analysis/substance flow analysis (MFA/SFA) for sustainable material management. Most MFA/SFA studies to date have mostly been concerned with one material or substance at a time, and the issue of mixing has seldom been examined (Rechberger and Graedel (2002) and Nakamura and Yamasue (2010) are rare exceptions).

Visual representation of flows is a vital component of any MFA/SFA study. The Sankey diagram, which has been a standard tool for visualizing MFA/SFA results, may become too complex for effective visualization as the number of processes under consideration increases. Such a case occurs when economy-wide inter-sector flows of materials are concerned. There is, therefore, a need for alternative methods for graphical visualization of MFA/SFA results.

With this background, this paper proposes a new tool for graphically visualizing the economy wide inter-sector flows of various types of materials which combines the hybrid input-output (IO) methods that we have developed. Nakamura et al. (2011) developed a hybrid IO method, named UPIOM (the unit physical input-output by materials), for identifying the physical IO flows of individual materials that are associated with the production of a unit of given product. The results of UPIOM are obtained in the form of as many inter-sector flow matrices as the number of material types being considered. Nakamura et al. (2011) also developed a tool for visually representing the calculated UPIOM for each type of material that makes use of triangulation of the inter-sector flow matrix based on degrees of fabrication.

Parts are embedded in a product, whilst the product is not embedded in the parts. By rearranging the sectors in a descending order of their degrees of fabrication, an IO matrix can be triangulated such that most elements in the upper triangular part are zeros (Leontief 1963, Charon and Hudry 2010). A sequence of sectors that triangulates an IO matrix can be obtained by solving a combinatorial optimization problem in which the sum of the elements in the upper triangular part of the matrix is minimized. Because an optimal

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solution for triangulating an IO matrix is not unique in most practical applications, special attention should be paid in comparing two or more IO tables in terms of the hierarchies of sectors based on the individual triangulation of those tables. The occurrence of quite different sequences of sectors as optimal solutions for those IO tables does not imply that the underlying hierarchical structures in those IO tables are also different. There may still be a common sequence that can triangulate those tables. Kondo (2010) proposed a mixed-integer program (MIP) for triangulating two IO tables in a consistent manner, which has been further generalized here to cope with the case involving more than two IO tables.

Having obtained a common sequence that triangulates a UPIOM for a set of different types of materials, an innovative diagram, named UPIOM Cube, is constructed that gives a three dimensional graphic representation of the flows of these materials among make and use sectors. This generalizes the two dimensional UPIOM diagram proposed by Nakamura et al. (2011) for a given material. Amongst other uses, UPIOM Cube will be particularly useful in identifying the likely source of quality degradation of metals due to the mixing with other metal species.

The developed method is applied to a body of hybrid input-output data for Japan consisting of around 400 producing sectors. The passenger car is chosen as the final product of concern because of the importance of the car manufacturing industry in the Japanese economy. Iron and copper were chosen as the materials of concern because iron is the largest content of a car and copper is a major tramp element in iron and steel scrap. The estimated UPIOM indicates that flows are not evenly distributed over all industry-sectors of the economy, but tends to be concentrated among a cluster of around 30 sectors. A UPIOM Cube diagram was obtained for the flows involving these 30 sectors.

2. Methods

2.1 UPIOM

Nakamura et al. (2011) developed UPIOM (the unit physical input-output by materials) as a hybrid IO method for MFA/SFA. We briefly explain it in this sub-section.

Let \mathbf{A} denote the input coefficient matrix, the (i, j) -element, A_{ij} , of which represents the amount of input i directly required to produce a unit of output j . The Leontief inverse is given by $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$, where \mathbf{I} refers to an identity matrix of suitable dimension.

Group the n industry sectors into three categories: n_P types of *products* (P), n_M types of *materials* (M) and n_R types of *resources* (R). Rearranging the sectors in that order, the input coefficient matrix \mathbf{A} can be rewritten as

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_{PP} & \mathbf{A}_{PM} & \mathbf{A}_{PR} \\ \mathbf{A}_{MP} & \mathbf{A}_{MM} & \mathbf{A}_{MR} \\ \mathbf{A}_{RP} & \mathbf{A}_{RM} & \mathbf{A}_{RR} \end{pmatrix}.$$

Materials (M) and *resources* (R) are measured in physical units (kilogram), whilst *products* (P) are measured in their own specific units such as monetary units, kilogram, or joule. Let \mathbf{L}_{MP} denote the (M, P) -part of the Leontief inverse. The (i, j) -element of \mathbf{L}_{MP} , $(\mathbf{L}_{MP})_{ij}$, gives the amount of material i which is directly and indirectly required to produce a unit of output j . Note that the inequality $\sum_{i \in M} (\mathbf{L}_{MP})_{ij} \geq \mu_j$ holds ($j \in P$), where μ_j refers to the mass of product j . This is because a portion of the total amount of various types of the required materials (the left-hand side) does not become the physical component of a product.

Nakamura et al. (2007) proposed a method to filter out specific inputs which do not become part of the physical components of products, from the input coefficient matrix. There are three groups of those inputs: inputs without mass, ancillary inputs, and process waste. A mass filter $\Phi = \text{diag}(\phi_1, \dots, \phi_n)$ removes inputs without mass: $\phi_i = 1$ if input i has its mass and $\phi_i = 0$ otherwise. A yield matrix $\Gamma = (\gamma_{ij})$ filters out ancillary inputs and process waste: $\gamma_{ij} = 0$ if input i is ancillary in the production process of product j , and

γ_{ij} refers to the yield ratio of input i in that process otherwise. Applying these filters, the input coefficient matrix \mathbf{A} can be transformed as

$$\tilde{\mathbf{A}} = \mathbf{\Gamma} \odot (\mathbf{\Phi}\mathbf{A}) = \begin{pmatrix} \tilde{\mathbf{A}}_{PP} & \tilde{\mathbf{A}}_{PM} & \tilde{\mathbf{A}}_{PR} \\ \tilde{\mathbf{A}}_{MP} & \tilde{\mathbf{A}}_{MM} & \tilde{\mathbf{A}}_{MR} \\ \tilde{\mathbf{A}}_{RP} & \tilde{\mathbf{A}}_{RM} & \tilde{\mathbf{A}}_{RR} \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{A}}_{PP} & \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{A}}_{MP} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{A}}_{RM} & \mathbf{0} \end{pmatrix},$$

where \odot refers to the Hadamard (element-by-element) product of two matrices of the same size. With this filtering, the (M, P) -part of the Leontief inverse $\tilde{\mathbf{L}} = (\mathbf{I} - \tilde{\mathbf{A}})^{-1}$ gives the material composition matrix:

$$\mathbf{C} = \tilde{\mathbf{L}}_{MP} = \tilde{\mathbf{A}}_{MP}(\mathbf{I} - \tilde{\mathbf{A}}_{PP})^{-1}.$$

That is, the (m, j) -element of \mathbf{C} gives the mass of material m that is contained in product j . Note that the equality $\sum_{i \in M} (\tilde{\mathbf{L}}_{MP})_{ij} = \sum_{i \in M} (\mathbf{C})_{ij} = \mu_j$ holds ($j \in P$). The hybrid IO method based on this filtering is called the waste input-output material flow analysis (WIO-MFA). See Nakamura et al. (2007) for details.

The UPIOM of material m associated with product j is defined as

$$\mathbf{U}(m, j) = \begin{pmatrix} \text{diag}((\mathbf{C})_{m \cdot}) \tilde{\mathbf{A}}_{PP} \\ (\tilde{\mathbf{A}}_{MP})_{m \cdot} \end{pmatrix} \text{diag}((\tilde{\mathbf{L}}_{PP})_{\cdot j}),$$

where $(\mathbf{C})_{m \cdot}$ and $(\tilde{\mathbf{A}}_{MP})_{m \cdot}$ are the m -th row of \mathbf{C} and $\tilde{\mathbf{A}}_{MP}$, respectively, and $(\tilde{\mathbf{L}}_{PP})_{\cdot j}$ is the j -th column of $\tilde{\mathbf{L}}_{PP}$. The (i, k) -element of $\mathbf{U}(m, j)$, $U_{ik}(m, j)$, refers to the mass of material m (e.g., iron) that is embedded in product j (e.g., car) as product k (e.g., car body) in the form of input i (e.g., coated steel). Define an augmented UPIOM, $\bar{\mathbf{U}}(m, j)$, by adding a column vector to it as

$$\bar{\mathbf{U}}(m, j) = \begin{pmatrix} \text{diag}((\mathbf{C})_{m \cdot}) \tilde{\mathbf{A}}_{PP} \text{diag}((\tilde{\mathbf{L}}_{PP})_{\cdot j}) & \mu_j \mathbf{e}_j \\ (\tilde{\mathbf{A}}_{MP})_{m \cdot} \text{diag}((\tilde{\mathbf{L}}_{PP})_{\cdot j}) & 0 \end{pmatrix},$$

where \mathbf{e}_j refers to the j -th unit vector, i.e., its j -th element is unity and the other elements are zeros. Note that this augmented UPIOM is of size $(n_p + 1) \times (n_p + 1)$. As Nakamura et al. (2011) showed, the following holds for the column and row sums of $\bar{\mathbf{U}}(m, j)$:

$$\sum_{i=1}^{n_p+1} \bar{U}_{ih}(m, j) = \sum_{k=1}^{n_p+1} \bar{U}_{hk}(m, j) \quad (h = 1, \dots, n_p).$$

The left-hand side represents the material m mass of product h produced for producing a unit of product j , while the right-hand side represents the material m mass of product h used for producing a unit of product j . Moreover, this equality implies that

$$\sum_{k=1}^{n_p} (\tilde{\mathbf{A}}_{MP})_{mk} (\tilde{\mathbf{L}}_{PP})_{kj} = \sum_{k=1}^{n_p} \bar{U}_{n_p+1,k}(m, j) = \sum_{i=1}^{n_p} \bar{U}_{i,n_p+1}(m, j) = \mu_j,$$

which means that the sum of direct material inputs is equal to the mass of the final product.

2.2 Triangulation of input-output matrices

Primary, secondary and tertiary sectors in IO tables are traditionally arranged in that order largely due to historical reasons, and to a lesser extent due to economic reasoning. If the sector classification of an IO table is detailed enough, the equality $A_{ij}A_{ji} = 0$ likely holds. For instance, a combustion engine (product i) is required to produce a car (product j), i.e., $A_{ij} > 0$, but not vice versa, i.e., $A_{ji} = 0$. Therefore, an IO table may be triangulated, in the sense that most elements in the upper triangular part are zeros, by rearranging the sectors (Chenery and Watanabe 1958, Leontief 1963, Simpson and Tsukui 1965). In a triangulated IO table, a sector that occurs at a higher point of the sequence can be interpreted to have a higher degree of fabrication than those that occur at lower points of the sequence. Nakamura et al. (2011) proposed a diagram to display

inter-sector material flows, which makes use of the triangulation of the flow matrix. Triangulation greatly improves the readability of graphically represented flow matrices. Because our main concern is to analyze various types of materials simultaneously, we utilize a triangulation method (Kondo, 2010), by which two matrices can be triangulated in a consistent manner.

Suppose for a while that our target is to triangulate an $n \times n$ matrix $\mathbf{Q} = (Q_{ij})$ that describes inter-dependence among sectors. For ease of exposition, we define the set of natural numbers referring to n sectors as $N = \{1, 2, \dots, n\}$. We then denote a permutation of n sectors by $\boldsymbol{\pi} = (\pi(1), \pi(2), \dots, \pi(n))$ and the set of all permutations of the sectors by Π . Given a permutation $\boldsymbol{\pi} \in \Pi$, let $\mathbf{Q}(\boldsymbol{\pi}) = (Q_{ij}(\boldsymbol{\pi}))$ denote the IO matrix in which the sectors are permuted according to $\boldsymbol{\pi}$, that is, $Q_{ij}(\boldsymbol{\pi}) = Q_{\pi(i)\pi(j)}$ ($i, j \in N$). The triangulation problem is formulated as a combinatorial optimization problem:

$$\text{TP1}(\mathbf{Q}) \begin{cases} \text{maximize} & \ell(\mathbf{Q}(\boldsymbol{\pi})) \\ \text{subject to} & \boldsymbol{\pi} \in \Pi, \end{cases}$$

where $\ell(\mathbf{Q}(\boldsymbol{\pi})) = \sum_{i>j} Q_{ij}(\boldsymbol{\pi})$, i.e., the sum of the elements in the lower triangular part of $\mathbf{Q}(\boldsymbol{\pi})$.

It is known in the literature that the triangulation problem TP1(\mathbf{Q}) is equivalent to the following integer program (deCani 1969, Grötschel et al. 1984):

$$\text{TP2}(\mathbf{Q}) \begin{cases} \text{maximize} & \sum_{i>j} \{(Q_{ij} - Q_{ji})X_{ij} + Q_{ji}\} \\ \text{subject to} & 0 \leq X_{ij} + X_{jk} - X_{ik} \leq 1 \quad (i < j < k; i, j, k \in N), \\ & X_{ij} \in \{0, 1\} \quad (i < j; i, j \in N). \end{cases}$$

The binary variable X_{ij} represents that $X_{ij} = 1$ if sector j precedes sector i , and $X_{ij} = 0$ otherwise. Given an optimal solution to TP2(\mathbf{Q}), the corresponding optimal permutation $\boldsymbol{\pi}$ can then be given by

$$\pi^{-1}(i) = \sum_{j=1}^n X_{ij} \quad (i \in N),$$

where $X_{ii} = 1$ ($i \in N$) and $X_{ij} = 1 - X_{ji}$ ($i \geq j; i, j \in N$).

Suppose that we have IO matrices representing the flow of n_M types of materials, $\mathbf{Q}^{(m)} = (Q_{ij}^{(m)})$ ($m \in M$). Suppose also that we have solved the integer program TP2($\mathbf{Q}^{(m)}$) for all the materials, and then obtained an optimal solution $\bar{X}_{ij}^{(m)}$, the corresponding optimal permutation $\bar{\boldsymbol{\pi}}^{(m)}$, and the optimal value $\bar{\ell}^{(m)}$ (maximized objective value) of the program. The following mixed integer program is used in this study to find a common sequence of sectors which triangulates IO matrices the best in the sense that the worst relative loss of triangularity of IO matrices is minimized.

$$\text{TP3}(S) \begin{cases} \text{minimize} & D \\ \text{subject to} & 0 \leq X_{ij} + X_{jk} - X_{ik} \leq 1 \quad (i < j < k; i, j, k \in N), \\ & \sum_{i>j} \{(Q_{ij}^{(m)} - Q_{ji}^{(m)})X_{ij} + Q_{ji}^{(m)}\} \geq (1 - D)\bar{\ell}^{(m)} \quad (m \in M), \\ & X_{ij} \in \{0, 1\} \quad (i < j; i, j \in N). \end{cases}$$

where $S = \{\mathbf{Q}^{(m)} | m \in M\}$. The first set of constraints on the binary variables in TP3 is the same as that in TP2. The second set of constraints in TP3 implies that the relative loss of triangularity is equal to or less than D , i.e., $1 - \ell(\mathbf{Q}^{(m)}(\boldsymbol{\pi})) / \bar{\ell}^{(m)} \leq D$, for each IO matrix, where $\boldsymbol{\pi}$ refers to the permutation that corresponds to X_{ij} 's in TP3.

3. Application of UPIOM Cube

The method explained in the previous section is applied to the same hybrid IO data that Nakamura et al. (2011) used, to obtain the UPIOM associated with passenger car production in Japan. The following four

types of ferrous and copper materials are considered: pig iron, iron and steel scrap (IS scrap), copper, and copper scrap. Then, we have $n_P = 401$ and $n_M = 4$ for this study.

The estimated UPIOM indicates that the flows are not evenly distributed, but tend to be concentrated among a cluster of around 30 sectors. We then choose sectors with substantial inter-sector material flow, according to the following condition:

Sector h is said to have substantial inter-sector material flow if $\bar{U}_{ih}(m, j) \geq \alpha \mu_j$ for some i and m or $\bar{U}_{hk}(m, j) \geq \alpha \mu_j$ for some k and m .

With $\alpha = 0.25\%$, the numbers of chosen sectors are 27, 24, 19 and 19 for pig iron, IS scrap, copper, and copper scrap, respectively, when the above condition is individually applied to each material. In contrast, 39 sectors are chosen when all four materials are simultaneously taken into account. Twenty-eight sectors are chosen for solely the ferrous metals, while 21 sectors are chosen for copper and copper scrap.

The mixed integer program TP3 was solved for the 39×39 flow matrices of the four materials. The relative loss of triangularity at optimality is very small: $D = 0.026\%$. Figure 1 shows the UPIOM Cube for pig iron, IS scrap, copper, and copper scrap involving 39 sectors with substantial inter-sector material flow. In the figure, the 21 sectors that have substantial flow of copper or copper scrap are gathered as compactly as possible, using additional constraints to the mixed integer program TP3, so that all of them are located within the top 22 sectors.

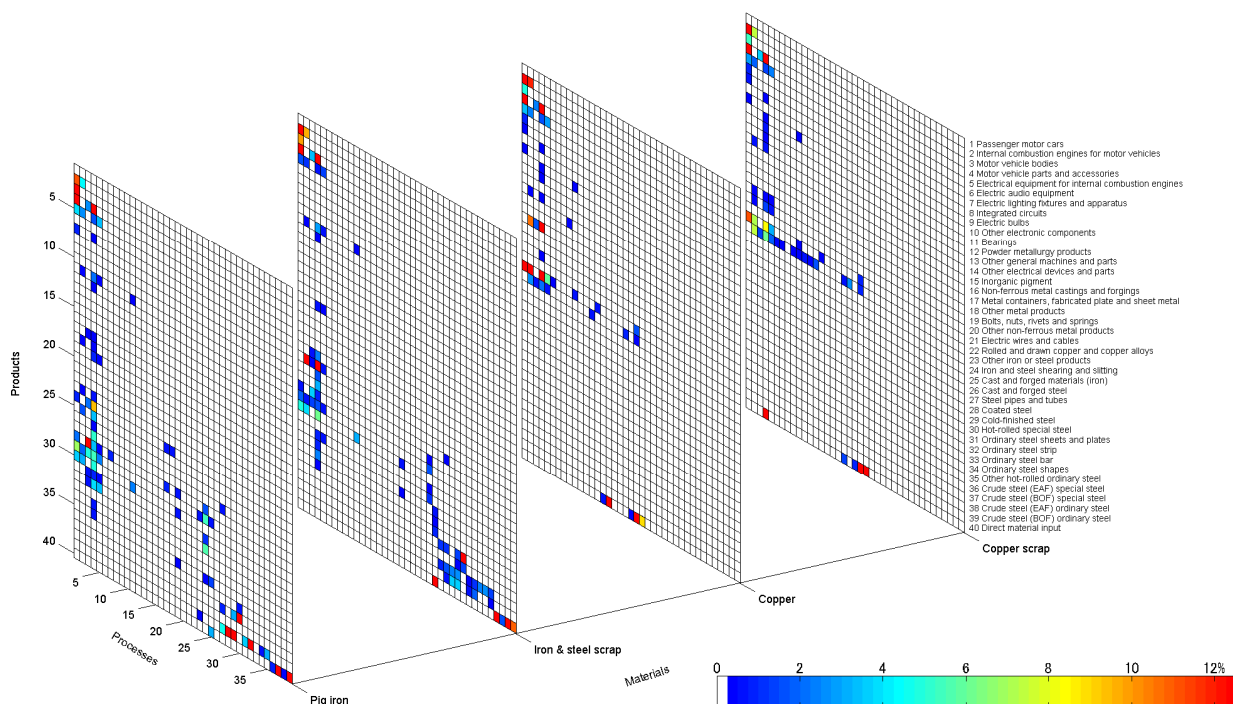


Figure 1
The UPIOM Cube of Pig Iron, Iron and Steel Scrap, Copper, and Copper Scrap
Associated with Passenger Car Production in Japan

Note: Each element represents the percentage of inter-sector material flow to the total mass embedded in a unit of a passenger car. Elements larger than 12.5% are displayed as if they are equal to 12.5%.

The UPIOM Cube has three axes, products, processes and materials, as the labels attached to the axes in Figure 1 show. A product-process plane corresponding to a material in the UPIOM Cube displays the inter-sector flows of that material. This is the UPIOM of that material developed by Nakamura et al. (2011). A product-material plane corresponding to a process depicts how much of each product is used by that process, and how much of each material constitutes those products. Moreover, a process-material plane corresponding to a product represents how much of each material in the form of that product goes into each process. A future direction for research is to develop a graphical interface to enable us to view a UPIOM Cube from whatever angle we want. Of the 39 sectors displayed in Figure 1, only 10 sectors have a significant simultaneous flow of both iron and copper. Attention to the flows into and out of these sectors will be of importance for identifying the likely source of quality degradation of iron and steel scrap due to mixing with copper. Another future direction for research is to incorporate more life-cycle aspects in this study, which includes purchase and use of cars, discard of ELV, and recycling of recovered scraps.

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